

# FIRST THEOREMS OF PREDICATE CALCULUS

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ABSTRACT. This module includes first proofs of predicate calculus theorems.

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## MODULE SPECIFICATION

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This module has the following specification:

Name: predtheo1  
Version: 1.00.00  
Rule version: 1.00.00  
Orgin: [http://www.meyling.com/principia/0\\_00\\_51/predtheo1\\_1.00.00\\_1.00.00.qedeq](http://www.meyling.com/principia/0_00_51/predtheo1_1.00.00_1.00.00.qedeq)

The following modules were used:

Name: propaxiom  
Version: 1.00.00  
Rule version: 1.00.00  
Orgin: [propaxiom\\_1.00.00\\_1.00.00.qedeq](#)  
pdf: [propaxiom\\_1.00.00\\_1.00.00.pdf](#)  
Name: predaxiom  
Version: 1.00.00  
Rule version: 1.00.00  
Orgin: [predaxiom\\_1.00.00\\_1.00.00.qedeq](#)  
pdf: [predaxiom\\_1.00.00\\_1.00.00.pdf](#)

First we prove a simple implication:

**Theorem 0.1.**

$$(\forall x(R(x)) \Rightarrow \neg\exists x(\neg R(x)))$$

*Proof.*

1	$(\exists x(R(x)) \Rightarrow R(y))$	add axiom axiom5
2	$(\exists x(\neg R(x)) \Rightarrow \neg R(y))$	replace $R(@S_1)$ by $\neg R(@S_1)$ in 1
3	$(\neg \exists x(\neg R(x)) \vee \neg R(y))$	use abbreviation impl in 2 at occurrence 1
4	$((P \vee Q) \Rightarrow (Q \vee P))$	add axiom axiom3
5	$((\neg \exists x(\neg R(x)) \vee Q) \Rightarrow (Q \vee \neg \exists x(\neg R(x))))$	replace $P$ by $\neg \exists x(\neg R(x))$ in 4
6	$((\neg \exists x(\neg R(x)) \vee \neg R(y)) \Rightarrow (\neg R(y) \vee \neg \exists x(\neg R(x))))$	replace $Q$ by $\neg R(y)$ in 5
7	$(\neg R(y) \vee \neg \exists x(\neg R(x)))$	MP with 3, 6
8	$(R(y) \Rightarrow \neg \exists x(\neg R(x)))$	reverse abbreviation impl in 7 at occurrence 1
9	$(\forall y(R(y)) \Rightarrow \neg \exists x(\neg R(x)))$	Particularize by $y$ in 8
10	$(\forall x(R(x)) \Rightarrow \neg \exists x(\neg R(x)))$	rename $y$ into $x$ in 9 at occurrence 1

□

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