

FIRST THEOREMS OF PROPOSITIONAL CALCULUS

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ABSTRACT. This module includes first proofs of propositional calculus theorems.

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MODULE SPECIFICATION

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This module has the following specification:

Name: proptheo1
Version: 1.00.00
Rule version: 1.00.00
Origin: http://www.meyling.com/principia/0_00_51/proptheo1_1.00.00_1.00.00.qedeq

The following modules were used:

Name: propaxiom
Version: 1.00.00
Rule version: 1.00.00
Origin: [propaxiom_1.00.00_1.00.00.qedeq](#)
pdf: [propaxiom_1.00.00_1.00.00.pdf](#)

First we prove a well known tautology:

Theorem 0.1.

$$(\neg P \vee P)$$

Proof.

1	$((P \Rightarrow Q) \Rightarrow ((A \vee P) \Rightarrow (A \vee Q)))$	add axiom axiom4
2	$((((P \vee P) \Rightarrow Q) \Rightarrow ((A \vee (P \vee P)) \Rightarrow (A \vee Q))))$	replace P by (P ∨ P) in 1
3	$((((P \vee P) \Rightarrow P) \Rightarrow ((A \vee (P \vee P)) \Rightarrow (A \vee P))))$	replace Q by P in 2
4	$((((P \vee P) \Rightarrow P) \Rightarrow ((\neg P \vee (P \vee P)) \Rightarrow (\neg P \vee P))))$	replace A by ¬P in 3

5	$((P \vee P) \Rightarrow P)$	add axiom axiom1
6	$((\neg P \vee (P \vee P)) \Rightarrow (\neg P \vee P))$	MP with 5, 4
7	$((P \Rightarrow (P \vee P)) \Rightarrow (\neg P \vee P))$	reverse abbreviation impl in 6 at occurrence 1
8	$(P \Rightarrow (P \vee Q))$	add axiom axiom2
9	$(P \Rightarrow (P \vee P))$	replace Q by P in 8
10	$(\neg P \vee P)$	MP with 9, 7

□

We just use our first sentence to get the second theorem:

Theorem 0.2.

$$(P \Rightarrow P)$$

Proof.

1	$(\neg P \vee P)$	add sentence theo1
2	$(P \Rightarrow P)$	reverse abbreviation impl in 1 at occurrence 1

□

And another use of the first theorem, to get the law of the excluded middle (tertium non datur):

Theorem 0.3.

$$(P \vee \neg P)$$

Proof.

1	$(\neg P \vee P)$	add sentence theo1
2	$((P \vee Q) \Rightarrow (Q \vee P))$	add axiom axiom3
3	$((\neg P \vee Q) \Rightarrow (Q \vee \neg P))$	replace P by $\neg P$ in 2
4	$((\neg P \vee P) \Rightarrow (P \vee \neg P))$	replace Q by P in 3
5	$(P \vee \neg P)$	MP with 1, 4

□

Also trivial is:

Theorem 0.4.

$$(P \Rightarrow \neg\neg P)$$

Proof.

1	$(P \vee \neg P)$	add sentence theo3
2	$(\neg P \vee \neg\neg P)$	replace P by $\neg P$ in 1
3	$(P \Rightarrow \neg\neg P)$	reverse abbreviation impl in 2 at occurrence 1

□

Three negations:

Theorem 0.5.

$$(P \vee \neg\neg\neg P)$$

Proof.

1	$(P \Rightarrow \neg\neg P)$	add sentence theo4
2	$((P \Rightarrow Q) \Rightarrow ((A \vee P) \Rightarrow (A \vee Q)))$	add axiom axiom4
3	$((P \Rightarrow \neg\neg P) \Rightarrow ((A \vee P) \Rightarrow (A \vee \neg\neg P)))$	replace Q by $\neg\neg P$ in 2
4	$((A \vee P) \Rightarrow (A \vee \neg\neg P))$	MP with 1, 3
5	$((A \vee \neg P) \Rightarrow (A \vee \neg\neg\neg P))$	replace P by $\neg P$ in 4
6	$((P \vee \neg P) \Rightarrow (P \vee \neg\neg\neg P))$	replace A by P in 5
7	$(P \vee \neg P)$	add sentence theo3
8	$(P \vee \neg\neg\neg P)$	MP with 7, 6

□

Now we could prove the reverse of Proposition 4:

Theorem 0.6.

$$(\neg\neg P \Rightarrow P)$$

Proof.

1	$(P \vee \neg\neg\neg P)$	add sentence theo5
2	$((P \vee Q) \Rightarrow (Q \vee P))$	add axiom axiom3
3	$((P \vee \neg\neg\neg P) \Rightarrow (\neg\neg\neg P \vee P))$	replace Q by $\neg\neg\neg P$ in 2
4	$(\neg\neg\neg P \vee P)$	MP with 1, 3
5	$(\neg\neg P \Rightarrow P)$	reverse abbreviation impl in 4 at occurrence 1

□

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